

# On the possibility of automation-induced stagnation

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## Abstract

We analyze the long-run growth effects of automation in the standard overlapping generations framework. We show that, in contrast to other neoclassical models of capital accumulation, automation does not promote growth but induces economic stagnation. The reason is that automation suppresses wages, which is the only source of investment in the overlapping generations framework.

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# 1 Introduction

Automation and its potential economic consequences have caught the attention of economists and the general public over the last few years (see, for example, The Economist, 2014). While some economists show how automation can lead to economic prosperity (Steigum, 2011; Acemoglu and Restrepo, 2015; Graetz and Michaels, 2015; Abeliansky and Prettner, 2017; Prettner, 2017), others are afraid that automation could (partly) be responsible for economic stagnation (Sachs and Kotlikoff, 2012; Benzell et al., 2015; Sachs et al., 2015). We contribute to this debate by showing that the long-run economic growth effects of automation crucially depend on the underlying framework used to describe the process of saving and investment. While the standard neoclassical growth models lead to remarkably similar predictions regarding the growth effects of changes in household's savings behavior, we show that they lead to diametrically opposed predictions of the effect of automation. On the one hand, models of automation based on Solow (1956), Ramsey (1928), Cass (1965), and Koopmans (1965), in which households save a part of wage income *and* a part of asset income, imply that automation has the potential to lead to perpetual long-run growth even without (exogenous or endogenous) technological progress. On the other hand, models of automation based on the overlapping generations (OLG) framework of Diamond (1965), in which households save *exclusively* out of wage income, imply economic stagnation in the face of automation. The reason for stagnation is that the accumulation of automation capital suppresses wages, which is the only source of investment in the OLG framework. Consequently, automation itself prevents the takeoff that would occur in neoclassical types of growth models in which also parts of asset income are (re-)invested.

## 2 The effects of automation in an OLG framework

Consider a household who lives for three time periods, youth, adulthood, and retirement. Children do not make any economic decisions and fulfill their needs via the consumption expenditures of their parents. Adults supply their available time on the labor market for the market clearing wage  $w_t$  and save for retirement. Retirees do not work and finance their consumption expenditures at old age out of their savings carried over from adulthood. The number of children is denoted by  $n$  such that the evolution of the population size is exogenous and given by  $N_{t+1} = (1 + n)N_t$ , where  $N_t$  refers to the size of the adult cohort at time  $t$ .

Following Diamond (1965), households derive utility from consumption in adulthood,  $c_{1,t}$ , and from consumption in retirement,  $c_{2,t+1}$ . Assuming that households discount the future at rate  $\rho$ , which implies a discount factor of  $\beta = 1/(1 + \rho)$ , the household's lifetime utility is given by

$$U_t = \log(c_{1,t}) + \beta \log(c_{2,t+1}). \tag{1}$$

Denoting the real interest rate on savings between time  $t$  and time  $t+1$  by  $r_{t+1}$ , the budget constraint of households is standard and given by

$$c_{1,t} + \frac{c_{2,t+1}}{1 + r_{t+1}} = w_t, \quad (2)$$

where the left-hand side refers to discounted lifetime consumption expenditures and the right-hand side to lifetime labor income. Solving the households' intertemporal optimization problem yields the consumption Euler equation

$$\frac{c_{2,t+1}}{c_{1,t}} = \beta(1 + r_{t+1}) \quad (3)$$

describing the optimal individual consumption growth path for a given interest rate and a given discount factor. From this expression and the budget constraint, optimal consumption and savings of adults follow as

$$c_{1,t} = \frac{1}{1 + \beta} w_t, \quad s_t = \frac{\beta}{1 + \beta} w_t. \quad (4)$$

There are three production factors: labor, which is supplied by adults, traditional physical capital in the form of machines, factory buildings, etc., which is an *imperfect* substitute for labor, and automation capital in the form of industrial robots, 3D printers, etc., which is, by definition, a *perfect* substitute for labor. Both types of capital are accumulated according to the investment decisions of households and a no-arbitrage relationship holds between the two investment vehicles.

The representative firm has access to a production technology as described by Prettnner (2017)

$$Y_t = K_t^\alpha (N_t + P_t)^{1-\alpha}, \quad (5)$$

where  $Y_t$  denotes aggregate output (GDP),  $K_t$  denotes the stock of traditional physical capital,  $P_t$  denotes the stock of automation capital, and  $\alpha \in (0, 1)$  is the elasticity of output with respect to traditional physical capital. We assume that there is perfect competition in goods and factor markets such that production factors are paid their marginal value products. Assuming that aggregate output is the numéraire, we get the following factor rewards

$$w_t \stackrel{!}{=} R_t^p = (1 - \alpha) \left( \frac{K_t}{N_t + P_t} \right)^\alpha, \quad (6)$$

$$R_t^k = \alpha \left( \frac{N_t + P_t}{K_t} \right)^{1-\alpha}, \quad (7)$$

where  $R_t^k$  is the rate of return on traditional physical capital and  $R_t^p$  is the rate of return on automation capital. We observe that, while an increase in traditional physical capital raises the wage rate, as in the standard Diamond (1965) model, an increase in automation capital has the opposite effect because automation competes with workers.

### 3 Equilibrium and main results

In an interior equilibrium, both savings vehicles have to deliver the same rate of return such that rational investors invest in both types of savings vehicles. Consequently, in any interior equilibrium, we have that  $R_t^k = R_t^p$  from which the following no-arbitrage relationship derives

$$P_t = \left( \frac{1 - \alpha}{\alpha} \right) K_t - N_t. \quad (8)$$

The intuition behind this expression is related to (6) and (7): a higher stock of traditional physical capital ( $K_t$ ) raises the rate of return on investment in automation capital ( $P_t$ ) and hence, *ceteris paribus* raises the stock of automation capital. By contrast, a larger cohort size of adults ( $N_t$ ) implies that there are more workers available such that the incentives for automation are lower (see also Abeliansky and Prettnner, 2017).

Plugging the no-arbitrage relationship (8) into the production function (5) yields an *AK*-type of technology in equilibrium

$$Y_t = \frac{1 - \alpha}{\alpha} K_t, \quad (9)$$

where  $A \equiv (1 - \alpha)/\alpha$ . As is well-known, such a production structure typically leads to perpetual growth because there are no diminishing returns with respect to the accumulation of physical capital (see, for example, Romer, 1986; Rebelo, 1991). As far as neoclassical models of automation that admit a representative household (Solow, 1956; Ramsey, 1928; Cass, 1965; Koopmans, 1965) are concerned, there is indeed the possibility of perpetual long-run growth (see Steigum, 2011; Prettnner, 2017). The intuition behind this result is that the availability of automation capital as a perfect substitute for labor prevents the diminishing returns from capital accumulation from kicking in and hence impedes the standard neoclassical convergence mechanism.

Since the economy is closed and both types of capital fully depreciate over the course of one generation, the aggregate stock of assets at time  $t + 1$  is determined by investment in period  $t$ . This implies that we have the following law of motion for the aggregate stock of assets

$$S_t = s_t N_t \stackrel{!}{=} K_{t+1} + P_{t+1} = \frac{\beta(1 - \alpha)}{1 + \beta} \left( \frac{K_t}{N_t + P_t} \right)^\alpha N_t. \quad (10)$$

In this case a competitive equilibrium can be defined as follows.

**Definition 1.** *A competitive equilibrium is a sequence  $\{K_t, P_t, c_{1,t}, c_{2,t}, R_t, R_t^k, R_t^p, w_t\}_{t=0}^\infty$ , such that  $\{R_t, R_t^k, R_t^p, w_t\}_{t=0}^\infty$  satisfy (6), (7), and  $R_t = R_t^k = R_t^p$ ,  $\{c_{1,t}, c_{2,t}\}_{t=0}^\infty$  satisfy (3) and (4),  $\{K_t, P_t\}_{t=0}^\infty$  satisfy (8) and (10), and  $\{N_t\}_{t=0}^\infty$  satisfies the population growth equation  $N_{t+1} = (1 + n)N_t$ .*

Dividing (10) by the size of the adult cohort  $N_{t+1}$  and plugging in the aggregate

production function (5) and the no-arbitrage condition (8), we arrive at the steady-state capital-labor ratio of the economy as given by

$$k = \alpha + \frac{(1 - \alpha)\alpha\beta}{(1 + n)(1 + \beta)(1/\alpha - 1)^\alpha}. \quad (11)$$

It is immediately clear that there is no growth in the capital-labor ratio. Consequently, GDP per capita stagnates and there is no potential for long-run economic growth. We summarize our main finding in the following proposition

**Proposition 1.** *In the Diamond (1965) model with automation, the production structure resembles the properties of an AK type of growth model. However, in contrast to other models of automation with this production structure, the economy is trapped in a stagnation equilibrium.*

This proposition implies that, in contrast to the standard neoclassical literature with a representative agent, there is the possibility of automation-induced stagnation in an OLG model. The reason is that investment is fully financed out of wage income, which is, at the same time, reduced by automation. In a sense, automation is therefore digging its own grave in the OLG model. Our result provides an explanation for stagnation due to automation as found by Sachs and Kotlikoff (2012), Benzell et al. (2015), and Sachs et al. (2015). However, we show that this result is an artifact of the underlying OLG framework such that it cannot be stated as a general economic regularity that pertains to automation.

## 4 Conclusions

We demonstrate that the possibility of automation-induced stagnation exists in the OLG model of Diamond (1965). The reason is that, in this framework, households exclusively save out of their labor income. By definition, however, automation competes with labor and depresses the wage rate and therefore labor income. This in turn reduces the savings and investment potential of households and prevents the economy from growing. Our result explains the numerical findings of Sachs and Kotlikoff (2012), Benzell et al. (2015), and Sachs et al. (2015), but also shows that their result is not generalizable to other models of capital accumulation.

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