Online Appendix

Price setting frequency and the Phillips curve

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A Model details

A.1 Price dispersion

Given the Calvo law of motion, price dispersion is a more complex process relative to the standard trend inflation NK model. The time-varying θ_t, can amplify or mute the non-monotonic behavior of price dispersion. In order to illustrate this point, consider the definition of relative price dispersion

$$s_t \equiv \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} di.$$

Under the Calvo pricing this can be expressed as

$$s_t = \frac{1}{P_t^{-\epsilon}} \left(\sum_{k=0}^{\infty} \theta_{t|t-k}(1-\theta_{t-k})(P^*_{i,t-k})^{-\epsilon} \right), \quad \text{where } \theta_{t|t-k} = \begin{cases} \Pi_{s=0}^{k-1}\theta_{t-s}, & \text{if} \quad k \geq 1, \\ 1, & \text{if} \quad k = 0, \end{cases}$$

or, recursively as

$$s_t = (1-\theta_t)(p_t^*)^{-\epsilon} + \theta_t \pi_t^{\epsilon} s_{t-1}.$$

From the above expression for s_t one can see that the time-varying Calvo share θ_t implies time-varying effects on price dispersion that can amplify or mute the non-monotonic effects of p_t^* and π_t on s_t. Suppose that a shock creates an incentive for firms to lower p_t^* and consequently leads to a decline in π_t. First, a lower p_t^* tends to raise s_t. Second, a lower π_t

tends to decrease s_t. A higher θ_t implies that less firms update to the new optimal price and therefore mutes the first effect and amplifies the second. The reverse is true for a lower θ_t. A similar reasoning applies to a shock that creates an incentive for firms to increase p_t^*.

A.2 Steady state

For $Y = 1$ and $\theta_t = \overline{\theta}$, the steady state of the model variables is determined by

$$
\begin{aligned}
(1+i) &= \overline{\pi}/\beta \\
w &= -\frac{\overline{\pi}\,(\epsilon-1)\left(\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}-1\right)}{\epsilon\left(\overline{\pi}-\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}\right)\left(\frac{\overline{\pi}^{\epsilon-1}\,\overline{\theta}-1}{\overline{\theta}-1}\right)^{\frac{1}{\epsilon-1}}} \\
p^* &= \frac{\epsilon}{\epsilon-1}\frac{\psi}{\phi} \\
\psi &= \frac{wY^{1-\sigma}}{1-\overline{\theta}\beta\overline{\pi}^{\epsilon}} \\
\phi &= \frac{Y^{1-\sigma}}{1-\overline{\theta}\beta\overline{\pi}^{\epsilon-1}} \\
p^f &= 1/\overline{\pi} \\
1 &= (\overline{\theta}\overline{\pi}^{\epsilon-1} + (1-\overline{\theta})p^{*\,1-\epsilon})^{\frac{1}{1-\epsilon}} \\
s &= \frac{(1-\overline{\theta})p^{*\ -\epsilon}}{(1-\overline{\theta}\overline{\pi}^{\epsilon})} \\
N &= Ys.
\end{aligned}
$$

A.3 The linearised New-Keynesian Phillips Curve

In order to understand how the Calvo law of motion affects the model dynamics in the linearised case, we linearise the NK Phillips curve around a trend inflation steady state as in Ascari and Sbordone (2014).[i] Throughout the linearisation, we assume $0 < \overline{\theta} < 1$ to avoid

[i] A hat $(\hat{\cdot})$ indicates that a variable is expressed in log-deviation from their steady state.

the empirically implausible polar cases $\overline{\theta} = \{0, 1\}$.

We start by linearising (5)

$$\hat{p}^*_t = \hat{\psi}_t - \hat{\phi}_t, \qquad \text{where} \tag{A.3.1}$$

$$\hat{\psi}_t = \hat{y}_t \left(\beta\, \overline{\pi}^{\epsilon}\, \overline{\theta} - 1\right) (\sigma - 1) - \hat{w}_t \left(\beta\, \overline{\pi}^{\epsilon}\, \overline{\theta} - 1\right) + \beta\, \overline{\pi}^{\epsilon}\, \overline{\theta}\, \mathbb{E}_t \hat{\psi}_{t+1} + \beta\, \overline{\pi}^{\epsilon}\, \overline{\theta}\, \mathbb{E}_t \hat{\theta}_{t+1} + \beta\, \epsilon\, \overline{\pi}^{\epsilon}\, \overline{\theta}\, \mathbb{E}_t \hat{\pi}_{t+1} \tag{A.3.2}$$

$$\hat{\phi}_t = \hat{y}_t \left(\beta\, \overline{\pi}^{\epsilon}\, \overline{\theta} - 1\right) (\sigma - 1) + \beta\, \overline{\pi}^{\epsilon-1}\, \overline{\theta}\, \mathbb{E}_t \hat{\phi}_{t+1} + \beta\, \overline{\pi}^{\epsilon-1}\, \overline{\theta}\, \mathbb{E}_t \hat{\theta}_{t+1} + \beta\, \overline{\pi}^{\epsilon-1}\, \overline{\theta}\, \mathbb{E}_t \hat{\pi}_{t+1} (\epsilon - 1). \tag{A.3.3}$$

Linearising (3) yields

$$\hat{p}^*_t = \frac{\overline{\theta}\, \hat{\theta}_t \left(\overline{\pi} - \overline{\pi}^{\epsilon}\right) + \overline{\pi}^{\epsilon}\, \overline{\theta}\, \hat{\pi}_t\, (\epsilon - 1) \left(\overline{\theta} - 1\right)}{(\epsilon - 1) \left(\overline{\theta} - 1\right) \left(\overline{\pi} - \overline{\pi}^{\epsilon}\, \overline{\theta}\right)}. \tag{A.3.4}$$

Then we substitute (A.3.4) into (A.3.1)

$$\hat{\psi}_t = \hat{\phi}_t + \frac{\overline{\theta}\, \hat{\theta}_t \left(\overline{\pi} - \overline{\pi}^{\epsilon}\right) + \overline{\pi}^{\epsilon}\, \overline{\theta}\, \hat{\pi}_t\, (\epsilon - 1) \left(\overline{\theta} - 1\right)}{(\epsilon - 1) \left(\overline{\theta} - 1\right) \left(\overline{\pi} - \overline{\pi}^{\epsilon}\, \overline{\theta}\right)}. \tag{A.3.5}$$

Next, equalize (A.3.5) and (A.3.2)

$$\begin{aligned} &\hat{\phi}_t + \frac{\overline{\theta}\, \hat{\theta}_t \left(\overline{\pi} - \overline{\pi}^{\epsilon}\right) + \overline{\pi}^{\epsilon}\, \overline{\theta}\, \hat{\pi}_t\, (\epsilon - 1) \left(\overline{\theta} - 1\right)}{(\epsilon - 1) \left(\overline{\theta} - 1\right) \left(\overline{\pi} - \overline{\pi}^{\epsilon}\, \overline{\theta}\right)} \\ &\quad = \hat{y}_t \left(\beta\, \overline{\pi}^{\epsilon}\, \overline{\theta} - 1\right) (\sigma - 1) - \hat{w}_t \left(\beta\, \overline{\pi}^{\epsilon}\, \overline{\theta} - 1\right) + \beta\, \overline{\pi}^{\epsilon}\, \overline{\theta}\, \mathbb{E}_t \hat{\theta}_{t+1} \\ &\qquad + \beta\, \overline{\pi}^{\epsilon}\, \overline{\theta} \left(\mathbb{E}_t \hat{\phi}_{t+1} + \frac{\overline{\theta}\, \mathbb{E}_t \hat{\theta}_{t+1} \left(\overline{\pi} - \overline{\pi}^{\epsilon}\right) + \overline{\pi}^{\epsilon}\, \overline{\theta}\, \hat{\pi}_{t+1}\, (\epsilon - 1) \left(\overline{\theta} - 1\right)}{(\epsilon - 1) \left(\overline{\theta} - 1\right) \left(\overline{\pi} - \overline{\pi}^{\epsilon}\, \overline{\theta}\right)} \right) + \beta\, \epsilon\, \overline{\pi}^{\epsilon}\, \overline{\theta}\, \hat{\pi}_{t+1}. \end{aligned} \tag{A.3.6}$$

Finally, we use (A.3.3) to eliminate $\hat{\phi}_t$

$$\begin{aligned}
&\hat{y}_t \left(\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}-1\right)(\sigma-1)+\beta\,\overline{\pi}^{\epsilon-1}\,\overline{\theta}\,\mathbb{E}_t\hat{\phi}_{t+1}+\beta\,\overline{\pi}^{\epsilon-1}\,\overline{\theta}\,\mathbb{E}_t\hat{\theta}_{t+1}\\
&\quad+\frac{\overline{\theta}\,\hat{\theta}_t\left(\overline{\pi}-\overline{\pi}^{\epsilon}\right)+\overline{\pi}^{\epsilon}\,\overline{\theta}\,\hat{\pi}_t\,(\epsilon-1)\left(\overline{\theta}-1\right)}{(\epsilon-1)\left(\overline{\theta}-1\right)\left(\overline{\pi}-\overline{\pi}^{\epsilon}\,\overline{\theta}\right)}\\
&\quad+\beta\,\overline{\pi}^{\epsilon-1}\,\overline{\theta}\,\hat{\pi}_{t+1}\,(\epsilon-1)=\hat{y}_t\left(\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}-1\right)(\sigma-1)-\hat{w}_t\left(\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}-1\right)+\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}\,\mathbb{E}_t\hat{\theta}_{t+1}\\
&\quad+\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}\left(\mathbb{E}_t\hat{\phi}_{t+1}+\frac{\overline{\theta}\,\mathbb{E}_t\hat{\theta}_{t+1}\left(\overline{\pi}-\overline{\pi}^{\epsilon}\right)+\overline{\pi}^{\epsilon}\,\overline{\theta}\,\hat{\pi}_{t+1}\,(\epsilon-1)\left(\overline{\theta}-1\right)}{(\epsilon-1)\left(\overline{\theta}-1\right)\left(\overline{\pi}-\overline{\pi}^{\epsilon}\,\overline{\theta}\right)}\right)+\beta\,\epsilon\,\overline{\pi}^{\epsilon}\,\overline{\theta}\,\hat{\pi}_{t+1}.
\end{aligned}\tag{A.3.7}$$

Rearranging and collecting terms yields

$$\hat{\pi}_t=\alpha_1\hat{w}_t+\alpha_2\mathbb{E}_t\hat{\pi}_{t+1}+\alpha_3\mathbb{E}_t\hat{\phi}_{t+1}+\alpha_4\hat{\theta}_t+\alpha_5\mathbb{E}_t\hat{\theta}_{t+1},$$

where $\alpha_1=\frac{\left(1-\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}\right)\left(\overline{\pi}^{1-\epsilon}-\overline{\theta}\right)}{\overline{\theta}}$, $\alpha_2=\beta\left(\overline{\pi}^{\epsilon}\,\overline{\theta}-\epsilon+\epsilon\,\overline{\pi}-\epsilon\,\overline{\pi}^{\epsilon}\,\overline{\theta}+1\right)+\frac{\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}\,(\epsilon-1)}{\overline{\pi}}$, $\alpha_3=-\frac{\overline{\pi}^{1-\epsilon}-1}{(\epsilon-1)\left(\overline{\theta}-1\right)}$, $\alpha_4=\beta\,\overline{\pi}-\beta-\frac{\beta\,\overline{\pi}^{\epsilon}}{\epsilon-1}+\frac{\beta\,\overline{\pi}}{\epsilon-1}-\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}+\frac{\beta\,\overline{\pi}^{\epsilon}\,\overline{\theta}}{\overline{\pi}}-\frac{\beta\,\overline{\pi}^{\epsilon}}{(\epsilon-1)\left(\overline{\theta}-1\right)}+\frac{\beta\,\overline{\pi}}{(\epsilon-1)\left(\overline{\theta}-1\right)}$, and , $\alpha_5=-\beta\left(\overline{\pi}^{\epsilon-1}\,\overline{\theta}-1\right)$ $(\overline{\pi}-1)$. We can distinguish two cases.

$\overline{\pi}=1$. The special case of zero trend inflation implies $\alpha_3=\alpha_4=\alpha_5=0$. Thus, we obtain the textbook NK Phillips curve with $\alpha_1=\frac{(1-\beta\overline{\theta})(1-\overline{\theta})}{\overline{\theta}}$ and $\alpha_2=\beta$.

As in the standard NK model, inflation $\hat{\pi}_t$ is positively linked to expected inflation $\mathbb{E}_t\hat{\pi}_{t+1}$ and marginal cost $\hat{w}_t$. Thus, in a first-order approximation, the effect of the time-varying price setting frequency simply cancels. Nonetheless, it is important to mention that, while considering a non-zero trend inflation steady state appears generally plausible in light of the positive inflation targets proclaimed by many central banks, it is essential for our linear estimation. Also note that there is no difference in the steady state price of a price re-setter and a non price re-setter, i.e., $\overline{p}^f=\overline{p}^*$.

$\overline{\pi}>1$. The general case considers positive trend inflation. Our assumptions imply that $\alpha_1,\alpha_2,\alpha_3>0$, i.e., as in a standard trend inflation model, inflation $\hat{\pi}_t$ is positively linked to expected inflation $\mathbb{E}_t\hat{\pi}_{t+1}$, marginal cost $\hat{w}_t$ and $\mathbb{E}_t\hat{\phi}_{t+1}$. The last two terms with current and expected Calvo share $\hat{\theta}_t$ emerge because of the Calvo law of motion. In addition, also $\mathbb{E}_t\hat{\phi}_{t+1}$ is potentially affected by the time-varying price setting frequency via (A.3.3). Note that $\alpha_5 > 0 > \alpha_4$ with $|\alpha_4| > |\alpha_5|$. Moreover, $|\alpha_4|$ and $|\alpha_5|$ are increasing in $\overline{\pi}$ as well as $\overline{\theta}$, i.e., the higher trend inflation or the lower the steady state price setting frequency, the stronger does inflation react to the changes in the actual and expected share of unchanged prices $\hat{\theta}_t$.

B The Calvo share in a large recession

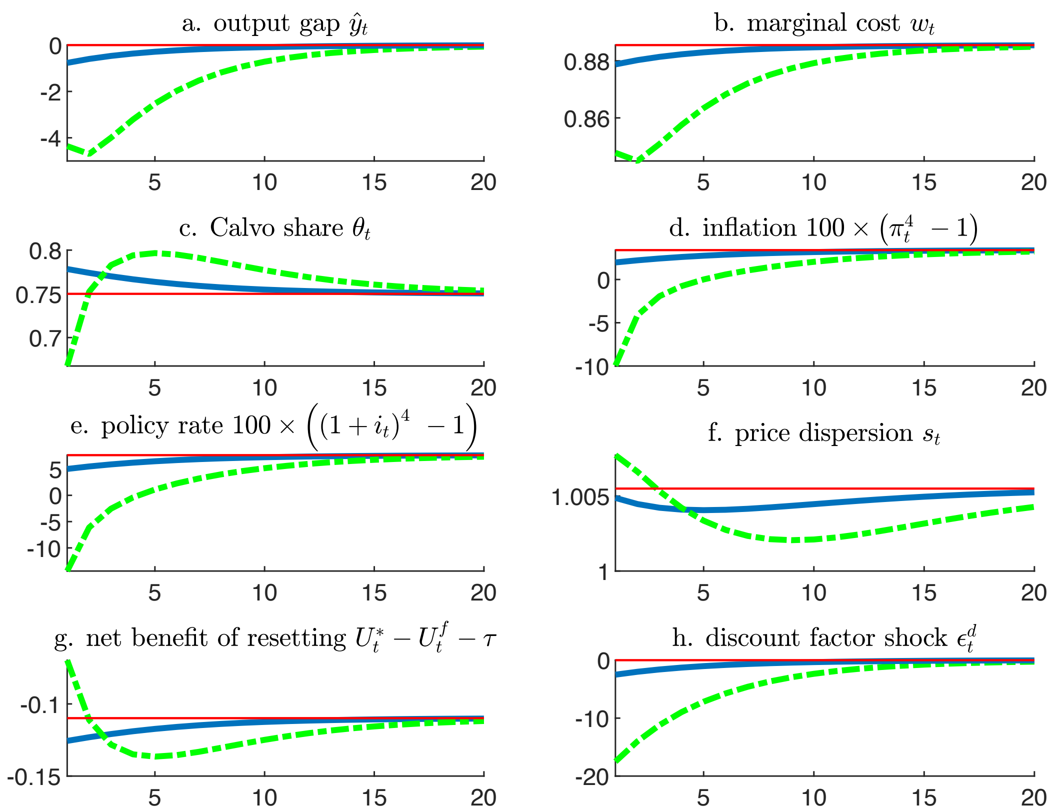

Figure B.1: Asymmetric impulse responses to a negative 2.5% (blue) and 17.5% (green) discount factor shock in the NK model. This choice implies a decline in the real interest rate of 0.5 (3.5 for the large shock) percent on impact. The persistence of the shock, $\rho_d = 0.8$ corresponds to a half-life of about 3 quarters in both cases. The unconditional standard deviation is 4.2 (29.2) percent. The negative shock implies an accumulated decline of real GDP of approximately 3 (22) percent over 7 quarters.